A MATHEMATICAL MODEL TO ELIMINATE POVERTY BY IMPLEMENTATION OF ZAKAT

Bio:

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Abstract:

Zakat is an important tool in maintaining a fair wealth distribution in the society. A new mathematical model to predict the wealth distribution among poor and rich is proposed based on the zakat. The model is a differential equation system which formulates the total belongings of rich subject to zakat and the total belongings of the poor who receive zakat. The global wealth transform between the two groups is expressed by the model. The aim in this study is to extrapolate and estimate wealth distribution among poor and rich starting from an initial state. The other aim in this study is to present in a rational manner (using mathematical models) the positive effects of fairer distribution of wealth in a society where zakat is practically applied. Three different scenarios are investigated: In the first one, the gaining of rich in a year is more than the zakat they pay, in the second, it is equal and in the last, less than the zakat they pay. In all scenarios, it is shown that zakat is an efficient tool to eliminate poverty in a society.

Key-words:

Zakat, Mathematical Modeling, Differential Equations

I. INTRODUCTION

Zakat is considered to be among the five pillars of Islam (Ganiyev and Umaraliev, 2020). Zakat has several meanings as a literary term such as growth, purification, blessing, development, and neatness (Razak et al. 2013). It is a must for believers (Wahid et al., 2017) to give a small fraction of their gaining in a yearly basis to the poor people. In the history, at the times of Prophet Muhammad (PBUM), the task of collection of zakat from the rich and distribution to the poor was maintained by the state (Wali, 2013). It was not a voluntary but a compulsory task to give zakat for the rich. Nowadays, in the societies heavily populated by Muslims, there are different implementations of zakat with respect to its collection. While some countries require it to be obligatory and the task of collection and distribution of zakat managed by the state, others treat it as a voluntary task left to individuals. In the countries where zakat is considered to be voluntary, it is usually collected and distributed by private and governmental associations on voluntary bases. Rich people may also choose to give their zakat directly to the poor. Turkey, being a secular country, uses the latter voluntary model.

Items covered by zakat are money, gold, silver, commercial assets, animal wealth, agricultural and animal products, mineral wealth and treasures, factories, earning assets and income from employment (Abdullahi, 2019). For people to be eligible to pay zakat, there is a minimum wealth limit called the amount of nisab. The rates of zakat from different items and the criteria for nisab were given by Norulazidah et al. (2010) (Tables 1 and 2 in that study).

The aim in this study is to model the wealth transform between the rich and poor via the zakat system so that the improvements in eliminating poverty can be shown in a rational mathematical manner. A fair wealth distribution is indeed a requirement for the peaceful existence in a society and it is vital not to make rich richer on a global basis (not on an individual basis) and the poor poorer. In the industrial revolution which started in the 19th century, there was a harsh application of capitalism which ended with many communist regimes all around the world. In fact, both systems went far to the extremes which resulted in widespread clashes and unhappiness within the people. Societies learned through painful experiences that a peaceful existence between the poor and rich can only be possible if a fraction of wealth is transferred from the wealthy to the needy people. In almost all countries, governments established social programs to support financially the people in need. Taxes taken from the rich are used to support poorer people via the social programs. Basically, the zakat and the social programs practiced by governments have one simple common goal, to transfer a fraction of the wealth from rich to poor. However, the details of both approaches vary much from each other when it comes to practically transferring the wealth. Comparing both systems is out of scope of this work and a detailed analysis is left to experts in the field. For managing our simpler aim, that is, to show the positive effects of zakat system on the wealth distribution, a mathematical model is proposed for the first time here in which the transfer of wealth is expressed via

an ordinary differential system. The zakat ratio and the earnings ratio are the basic parameters in the system together with the other parameters of interest. The model is solved for the specific cases of earnings ratio (subject to zakat) greater than the zakat ratio, ratios being equal and the earnings ratio being less than the zakat ratio. Numerical simulations reveal that in all three cases, the poverty in a society can be eliminated with the implementation of zakat.

Although the main aim of zakat is to eliminate poverty by transferring wealth from rich to poor, the zakat money can also be spend for other purposes such as religious activities, sponsoring the employees who collect zakat, to eliminate the debts of people, to travellers in need of money etc. However, the main portion of the collected zakat is always transferred to poor which is our focus in this study.

II. LITERATURE REVIEW

The influence of zakat on economic issues such as poverty reduction, economic growth, portfolio modelling, capital accumulation was discussed by mathematical and computational techniques. A mathematical model in the form of differential equation system was proposed by Subhan (2018). The model was based on determining the number of individuals who pay and receive zakat in a society. It is found that if zakat is managed in an optimal manner, the wealth equilibrium can be achieved. For a similar mathematical model in the form of differential equations and similar conclusions, see Putri (2017). The voluntary payment of zakat is modelled using both warm-glow and social custom frameworks by Norulazidah et al. (2010). Effect of zakat on capital accumulation and growth was investigated in that study. The efficiency of three zakat institutions were analyzed by employment of a computational technique, namely the data envelopment analysis (Rusydiana & Al-Farisi, 2016). The effect of zakat on the economic development was analyzed by the Analytical Networking Process Method (Hasbi & Widayanti, 2022). The positive influence of zakat on the economic growth was discussed using the data from Indonesia and evaluating the data by statistical methods (Rizal & Adibah, 2022). According to Powel (2010), there is a correlation between zakat and higher wealth and economic equality. In order to make zakat collection more effective, operations research techniques were employed and the positive effects of zakat on the economy was discussed (Abdullahi, 2019). For portfolio modelling, the study due to Subekti et al. (2022) highlights two mathematical models that consider the concept of zakat reduction and purification factors in the Indonesian stock market. Using a second-degree equation of Fourier series, a mathematical equation is designed to predict the amounts of Khums and potential zakat in the economy of Iran (Namdar et al., 2021). The algebraic mathematical model presented by Selim & Farooq (2020) shows how to transform a zakat receiver into a zakat payer by alleviation of poverty. An agent based computational model was used to study positive effect of zakat on economic growth (Putriani et al., 2020). Collection and distribution of zakat was expressed by a quadratic,

exponential and discrete Malthusian growth model and it is stated that the quadratic model is the best (Razak et al., 2013).

In the work mentioned above, various mathematical and computational techniques were employed to understand the effects of zakat on the society. The differential equation model presented here that describes the transfer of wealth from rich to the poor via the zakat system appears to be new. Previous differential equation models considered the number of individuals and their variation in numbers in a zakat system whereas, in our model, the global wealth transformation can be predicted. All previous studies point to the positive effects of zakat on the economy and wealth distribution. Using a new mathematical model, the same conclusions are verified in this study also. The time to reach a society where poverty is removed can also be predicted by the model.

III. METHODOLOGY

The method used in this work is to model mathematically the wealth transfer from rich to poor via the zakat system. For this purpose, a continuous model in the form of an ordinary differential equation system is derived. The problem is an initial value problem in which the total belongings and wealth of rich and poor start from an unequal state. The model predicts the transfer of wealth from rich to poor within time via the zakat system. Exact solutions are available without resort to purely numerical techniques. The exact solutions found are interpreted with numerical evaluations and usage of graphical representations.

First, a mathematical model in the form of a differential equation system will be derived under some simplified assumptions. Assume that r is the total money and zakated property owned by rich from which zakat has to be deduced. Assume that p is the total money and property owned by the poor. The aim is to develop a mathematical model expressing the change in years of the wealth between rich and poor so that a society with a healthy wealth distribution can be achieved. The time to eliminate poverty in a society can also be predicted by the model.

The rate of change of the money and zakated property of rich in time is directly proportional to their belongings and decreases by the zakat given

$$dr/dt = \beta r - \alpha r \qquad\qquad (1)$$

where β is the earnings ratio and α is the zakat ratio. For most of the items that is subject to zakat, the ratio is $1/40 = 0.025$.

The poor will get this zakat and one may assume that there is a constant rate of growth μ for their belongings. Hence

$$dp/dt = \alpha r + \mu \qquad\qquad (2)$$

Since the poor do not usually involve in a commercial activity and have salary based constant earnings, μ is not a fraction of their belongings, rather a constant rate in time. The initial belongings of rich and poor are

$$r(0)=r_0 , \qquad p(0)=p_0 \tag{3}$$

respectively which are the initial conditions for the differential equation system. Solving (1) with the corresponding initial condition yields

$$r(t)=r_0\exp[(\beta-\alpha)t] \tag{4}$$

which upon substitution into (2), integrating and applying the initial condition yields

$$p(t)=\alpha r_0 [\exp((\beta-\alpha)t)-1] /(\beta-\alpha)+\mu t+p_0 \tag{5}$$

Three distinct cases exist:

i)$\beta > \alpha$

For this case, the rich earn more than they give zakat. Assume that g_0 is the richness limit for which no poor to give zakat exists. Hence the belongings of rich grow exponentially until the belongings of poor reach the richness limit g_0

$$g_0=\alpha r_0[\exp((\beta-\alpha)t_1)-1] /(\beta-\alpha)+\mu t_1+p_0 \tag{6}$$

where t_1 is the time when the poor reach the richness limit. Given the quantities g_0, r_0, p_0, α, β and μ, the time t_1 cannot be solved explicitly and numerical root finding techniques are necessary for the solution.

The earnings of rich from t=0 to t=t_1 will be $r_0=\exp[(\beta-\alpha)t]$. But after t=$t_1$, there is nobody to receive zakat and hence α=0. If equation (1) with α=0 is solved with the initial condition

$$r(t_1)=r_0\exp[(\beta-\alpha)t_1] \tag{7}$$

the solution would be

$$r(t)=r_0\exp[\beta t-\alpha t_1] \tag{8}$$

To express the solution in the whole domain of interest, a gamma interval function is defined

$$\gamma[a,b)(t)=1 \text{ if } a\le t<b ; \qquad \gamma[a,b)(t)=0 \text{ if } t<a \text{ or } t\ge b \tag{9}$$

In terms of the gamma interval function, the solution valid throughout the whole domain is

$$r(t)=r_0\exp[(\beta-\alpha)t]\gamma[0,t_1)(t)+r_0\exp[\beta t-\alpha t_1]\gamma[t_1,\infty)(t) \tag{10}$$

A similar analysis gives the belongings of the poor in the whole domain of interest

$$p(t)=\{\alpha r_0[\exp((\beta-\alpha)t)-1]/(\beta-\alpha)+\mu t+p_0\}\gamma[0,t_1)(t)+(g_0+\mu(t-t_1))\gamma[t_1,\infty)(t) \tag{11}$$

where at t=t_1, p=g_0.

ii)$\beta < \alpha$

For this case, since the earnings of rich are less that the zakat they give, there will be an exponential decrease in their belongings. If one assumes that, when the poor reach the limit of richness g_0, r is higher than g_0, which is likely to occur in real cases, then the solutions (10) and (11) are still valid for this case.

iii) $\beta = \alpha$

For this very special case, the rich give all their earnings as zakat to the poor. This is a degenerate case to be treated. Equation (1) with $\beta=\alpha$ reads

$$dr/dt=0 \tag{12}$$

the solution of which is

$$r(t)=r_0 \tag{13}$$

subject to the initial condition. From (2)

$$dp/dt=\alpha r_0+\mu \tag{14}$$

with a linear solution of

$$p=(\alpha r_0+\mu)t+p_0 \quad . \tag{15}$$

The solutions are valid up to $t=t_1$ when the poor reaches the richness limit $p=g_0$. From (15)

$$g_0=(\alpha r_0+\mu)t_1+p_0 \tag{16}$$

and contrary to the previous case, t_1 can now be explicitly solved

$$t_1=(g_0-p_0)/(\alpha r_0+\mu) \quad . \tag{17}$$

When $t \geq t_1$, there is nobody to give zakat and $\alpha = 0$. The solutions would then be

$$r(t)=r_0\exp[\beta(t-t_1)] \ , \quad p(t)=g_0+\mu(t-t_1) \tag{18}$$

Using the gamma interval functions, the solutions valid throughout the whole domain can be expressed as

$$r(t)=r_0\gamma[0,t_1)(t)+r_0\exp[\beta(t-t_1)] \ \gamma[t_1,\infty)(t) \tag{19}$$

$$p(t)=[(\alpha r_0+\mu)t+p_0] \ \gamma[0,t_1)(t)+[g_0+\mu(t-t_1)] \ \gamma[t_1,\infty)(t) \quad . \tag{20}$$

IV. RESULTS AND ANALYSIS

Numerical results are presented in this section. Due to lack of data, without loss of generality, some normalized hypothetical rational data are taken in the simulations. In all calculations

$$r_0=100, \ \ p_0=20, \ \ g_0=50, \ \ \alpha=0.025, \ \ \mu=1 \tag{21}$$

are taken. The zakat ratio α is taken as 1/40 which is the most common rate in items considered to be subject to zakat. The three distinct cases given in the previous section are treated numerically.

In Figure 1, β=0.05 is taken which states that the earnings of rich are larger than the zakat ratio. Newton-Raphson algorithm (Pakdemirli & Boyacı, 2007) is employed in finding roots of algebraic equations. For an algebraic equation $f(x)=0$, the Newton-Raphson algorithm converges to the root through the iteration algorithm

$$x_{n+1}=x_n+f(x_n)/ f'(x_n), \quad n=0,1,2,\dots . \tag{22}$$

In our specific case, the Newton-Raphson algorithm yields $t_1=7.9654$ for the time the poor reaches the richness limit of $g_0=50$.
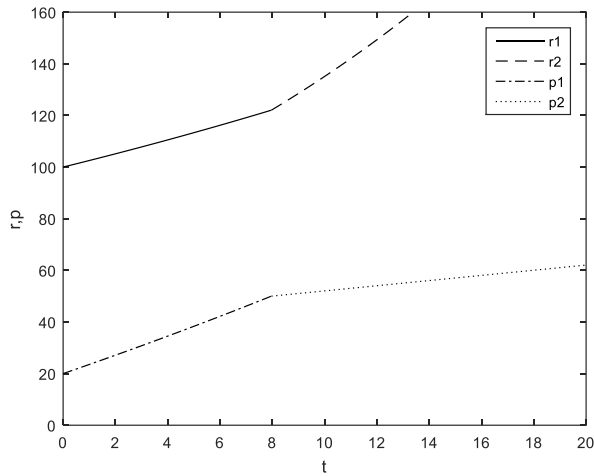

Figure 1. The belongings of rich and poor when the rate of earnings is higher than the rate of zakat

From the figure, r_1 represents the belongings of rich subject to zakat and r_2 corresponds to the earnings without paying zakat since there is nobody left in the society to receive zakat. As can be seen from the figure, the rate of earnings increased after there remains no zakat responsibility. In the meantime, the poor first reaches to the richness limit by taking zakat and their self-earnings. After they reach the limit, the rate of increase of their belongings are lower since they no longer receive zakat. Most of the curves are exponential in nature but looks linear in the Figure. This is due to the small fractions in the arguments of exponential functions which makes the first linear term dominant in the Taylor expansion compared to the higher order nonlinear terms.

In Figure 2, $\beta=0.025$ is taken which states that the earnings of rich are equal to the zakat ratio. From (17), $t_1=8.5714$ which is the time the poor reaches the richness limit.
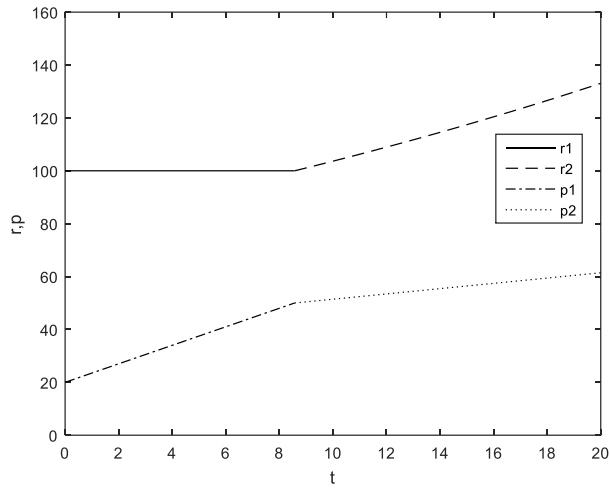

Figure 2. The belongings of rich and poor when the rate of earnings is equal to the rate of zakat

As can be seen from the figure, the belongings of the rich remain constant since they give all their earnings as zakat but then after the poor reaches the richness limit, their belongings increase exponentially as depicted in solution (19). In the meantime, the belongings of the poor increase linearly both before t_1 and after t_1. The slope is less after t_1 because they no longer receive zakat.

The last case is given in Figure 3, where $\beta=0.015$ is taken which states that the earnings of rich are less than the zakat ratio. Newton Raphson algorithm yields $t_1=8.8426$ for the time the poor reaches the richness limit of $g_0=50$. Solutions (10) and (11) describe again this situation.
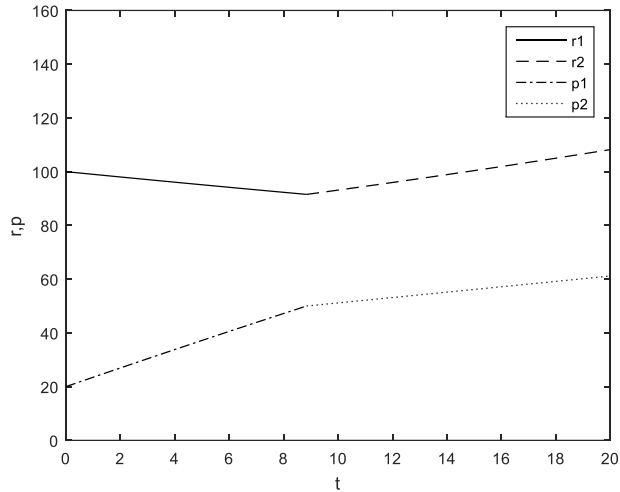
Figure 3. The belongings of rich and poor when the rate of earnings is less than the rate of zakat

The belongings of the rich first decrease because they pay more zakat than their earnings, but after the poor riches the richness limit, they do not have a requirement to pay zakat and their earnings start rising. Similar trends are observed in all the figures for the poor people.

The main result in the mathematical analysis is that poverty can be eliminated via a zakat system and a closed society can reach a level where there are no people left to give zakat. The time to reach this level is also predicted by the model. In all three cases (Rate of earnings of rich being higher, equal and less than the zakat rate), the model results in a society with poverty eliminated in the long run.

The simulations in this study assumes a closed society which is a more possible case in the past. In the time of Umar Ibn Abdul Aziz, caliphate of Umayyads, there were no poor left in the society to receive zakat (Putriani et al., 2020). Nowadays, for a heavily populated Muslim society, if there remains nobody to give zakat, then a Muslim can always sent his/her zakat to another country and may feel himself responsible to aid others abroad. Zakat therefore can also be used as a means of transfer of wealth from richer countries to poorer countries.

The analysis presented here can easily be adapted to real data if such data is available or at least estimated roughly for a given society. One obstacle to reach such data is that even for individuals, the calculation of zakat may become complex because of the several different items (money, gold, silver, commercial assets, animal wealth, agricultural and animal products, mineral wealth and treasures, factories, earning assets and income from employment) to be included in the calculations. Another obstacle is the voluntary

involvement of rich people to zakat in many countries and the lack of a government organization to collect the zakat. Our analysis is more applicable for countries where zakat is obligatory and the government takes the task of collecting and distributing it.

V. CONCLUSION AND RECOMMENDATION

In this study, a mathematical model is proposed for the first time to predict wealth transform between the rich and the poor in favor of the poor. The time to reach a level where poverty is totally eliminated from the society can also be determined by the model. A continuous model in the form of a differential equation is suggested. By the aid of mathematical solutions and numerical simulations, it is shown that zakat is an effective tool to eliminate poverty in a heavily populated Muslim country. This conclusion is in parallel with the findings of previous mentioned work that zakat implementation has positive effects on the economy and wealth distribution.

The model needs real data for extrapolating the wealth distribution in a country which can be officially demanded from the governments. The related parameters should be inserted into the model to predict the time when poverty can be eliminated from the society.

Further studies and improvements can be possible for the present work. The simple model proposed can be developed in a number of ways. Instead of a constant earning rate, a time dependent variable rate with a prescribed function of time can be taken. The numerical calculations can be repeated with real data. Instead of a continuous model, an equivalent discrete model can also be proposed.

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